

# Exploring Softly Broken SUSY Theories via Grassmannian Taylor Expansion

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## Abstract

We demonstrate that soft SUSY breaking introduced via replacement of the couplings of a rigid theory by spurion superfields has far reaching consequences. Substituting these modified couplings into renormalization constants, RG equations, solutions to these equations, fixed points, finiteness conditions, etc., one can get corresponding relations for the soft terms by a simple Taylor expansion over the Grassmannian variables. This way one can get new solutions of the RG equations. Some examples including the MSSM, SUSY GUTs and the N=2 Seiberg-Witten model are given.

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# 1 Introduction

In a recent paper [1], which is based on the previous publications [2, 3] we have shown that renormalizations in a softly broken SUSY theory follow from those of an unbroken SUSY theory and can be performed in the following straightforward way:

*One takes renormalization constants of a rigid theory, calculated in some massless scheme, substitutes instead of the rigid couplings (gauge and Yukawa) their modified expressions, which depend on a Grassmannian variable, and expand over this variable.*

This gives renormalization constants for the soft terms. Differentiating them with respect to a scale one can find corresponding renormalization group equations.

Thus the soft term renormalizations are not independent but can be calculated from the known renormalizations of a rigid theory with the help of the differential operators. Explicit form of these operators has been found in a general case and in some particular models like SUSY GUTs or the MSSM [1]. The same expressions were obtained also in ref. [4].

In this letter we demonstrate that this procedure works at all stages. One can make the above mentioned substitution on the level of the renormalization constants, RG equations, solutions to these equations, approximate solutions, fixed points, finiteness conditions, etc. Expanding then over a Grassmannian variable one obtains corresponding expressions for the soft terms. This way one can get new solutions of the RG equations and explore their asymptotics, or approximate solutions, or find their stability properties, starting from the known expressions for a rigid theory.

Below we give some examples and in particular consider the MSSM with low  $\tan \beta$ , where analytical solutions are known. We show how one can easily obtain solutions to the RG equations for the soft mass terms much simpler than are known in the literature. In a finite SUSY GUT finiteness conditions for the soft terms appear as a trivial consequence of a finiteness of a rigid theory. Another example is the N=2 SUSY model, where the exact (non-perturbative) Seiberg-Witten solution is known. Here one can extend the S-W solution to the soft terms.

## 2 Soft SUSY Breaking and Renormalization

Consider an arbitrary  $N = 1$  SUSY gauge theory with unbroken SUSY. The Lagrangian of a rigid theory is given by

$$\begin{aligned} \mathcal{L}_{rigid} = & \int d^2\theta \frac{1}{4g^2} \text{Tr} W^\alpha W_\alpha + \int d^2\bar{\theta} \frac{1}{4g^2} \text{Tr} \bar{W}^\alpha \bar{W}_\alpha. \\ & + \int d^2\theta d^2\bar{\theta} \bar{\Phi}^i (e^V)_i^j \Phi_j + \int d^2\theta \mathcal{W} + \int d^2\bar{\theta} \bar{\mathcal{W}}, \end{aligned} \quad (1)$$

where  $W^\alpha$  is the field strength chiral superfield and the superpotential  $\mathcal{W}$  has the form

$$\mathcal{W} = \frac{1}{6} \lambda^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M^{ij} \Phi_i \Phi_j. \quad (2)$$

To perform the SUSY breaking, which satisfies the requirement of "softness", one can introduce a gaugino mass term as well as cubic and quadratic interactions of scalar superpartners of the matter fields [2]

$$-\mathcal{L}_{soft-breaking} = \left[ \frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + h.c. \right] + (m^2)_j^i\phi_i^*\phi^j, \quad (3)$$

where  $\lambda$  is the gaugino field and  $\phi_i$  is the lower component of the chiral matter superfield.

One can rewrite the Lagrangian (3) in terms of N=1 superfields introducing the external spurion superfields [2]  $\eta = \theta^2$  and  $\bar{\eta} = \bar{\theta}^2$ , where  $\theta$  and  $\bar{\theta}$  are Grassmannian parameters, as [3]

$$\begin{aligned} \mathcal{L}_{soft} &= \int d^2\theta \frac{1}{4g^2}(1 - 2M\theta^2)\text{Tr}W^\alpha W_\alpha + \int d^2\bar{\theta} \frac{1}{4g^2}(1 - 2\bar{M}\bar{\theta}^2)\text{Tr}\bar{W}^\alpha \bar{W}_\alpha. \\ &+ \int d^2\theta d^2\bar{\theta} \bar{\Phi}^i(\delta_i^k - (m^2)_i^k\eta\bar{\eta})(e^V)_k^j\Phi_j \\ &+ \int d^2\theta \left[ \frac{1}{6}(\lambda^{ijk} - A^{ijk}\eta)\Phi_i\Phi_j\Phi_k + \frac{1}{2}(M^{ij} - B^{ij}\eta)\Phi_i\Phi_j \right] + h.c. \end{aligned} \quad (4)$$

Comparing eqs.(1) and (4) one can see that eq.(4) is equivalent to (1) with modification of the rigid couplings  $g^2$ ,  $\lambda^{ijk}$  and  $M^{ij}$ , so that they become external superfields dependent on Grassmannian parameters  $\theta^2$  and  $\bar{\theta}^2$ . The scalar mass term  $m^2\eta\bar{\eta}$  modifies fields  $\Phi$  and  $\bar{\Phi}$ . These modifications of the couplings and fields are valid not only for the classical Lagrangian but also for the quantum one.<sup>1</sup> As has been shown in Ref. [1] the following statement is valid:

*If a rigid theory (1, 2) is renormalized via introduction of renormalization constants  $Z_i$ , defined within some minimal subtraction massless scheme, then a softly broken theory (4) is renormalized via introduction of renormalization superfields  $\tilde{Z}_i$  which are related to  $Z_i$  by the coupling constants redefinition*

$$\tilde{Z}_i(g^2, \lambda, \bar{\lambda}) = Z_i(\tilde{g}^2, \tilde{\lambda}, \tilde{\bar{\lambda}}), \quad (5)$$

where the redefined couplings are

$$\tilde{g}^2 = g^2(1 + M\eta + \bar{M}\bar{\eta} + 2M\bar{M}\eta\bar{\eta}), \quad \eta = \theta^2, \quad \bar{\eta} = \bar{\theta}^2, \quad (6)$$

$$\tilde{\lambda}^{ijk} = \lambda^{ijk} - A^{ijk}\eta + \frac{1}{2}(\lambda^{njk}(m^2)_n^i + \lambda^{ink}(m^2)_n^j + \lambda^{ijn}(m^2)_n^k)\eta\bar{\eta}, \quad (7)$$

$$\tilde{\bar{\lambda}}_{ijk} = \bar{\lambda}_{ijk} - \bar{A}_{ijk}\bar{\eta} + \frac{1}{2}(\bar{\lambda}_{njk}(m^2)_i^n + \bar{\lambda}_{ink}(m^2)_j^n + \bar{\lambda}_{ijn}(m^2)_k^n)\eta\bar{\eta}. \quad (8)$$

Thus a softly broken SUSY gauge theory is equivalent to an unbroken one in external spurion superfield as far as the renormalization properties are concerned. Substitutions (6-8) can be made not only in the renormalization constants, but at every stage of the

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<sup>1</sup>Throughout the paper the existence of some SUSY invariant regularization is assumed.

renormalization procedure, since the RG functions and RG equations are derived from renormalization constants applying the differential operators. The key point is that one can consider an unbroken theory in external superfield which is equivalent to replacing of the couplings by external superfields according to eqs.(6-8). Then one can expand over Grassmannian parameters.

In what follows we would like to simplify the notations and consider numerical rather than tensorial couplings. When group structure and field content of the model are fixed, one has a set of gauge  $\{g_i\}$  and Yukawa  $\{y_k\}$  couplings. It is useful to consider the following rigid parameters

$$\alpha_i \equiv \frac{g_i^2}{16\pi^2}, \quad Y_k \equiv \frac{y_k^2}{16\pi^2}.$$

Then eqs.(6-8) look like

$$\tilde{\alpha}_i = \alpha_i(1 + M_i\eta + \bar{M}_i\bar{\eta} + 2M_i\bar{M}_i\eta\bar{\eta}), \quad (9)$$

$$\tilde{Y}_k = Y_k(1 + A_k\eta + \bar{A}_k\bar{\eta} + (A_k\bar{A}_k + \Sigma_k)\eta\bar{\eta}), \quad (10)$$

where to standardize the notations we have redefined parameter A:  $A \rightarrow Ay$  in a usual way and have changed the sign of A to match it with the gauge soft terms. Here  $\Sigma_k$  stands for a sum of  $m^2$  soft terms, one for each leg in the Yukawa vertex.

Now the RG equation for a rigid theory can be written in a universal form

$$\dot{a}_i = a_i\gamma_i(a), \quad a_i = \{\alpha_i, Y_k\}, \quad (11)$$

where  $\gamma_i(a)$  stands for a sum of corresponding anomalous dimensions. In the same notation the soft terms (9,10) take the form

$$\tilde{a}_i = a_i(1 + m_i\eta + \bar{m}_i\bar{\eta} + S_i\eta\bar{\eta}), \quad (12)$$

where  $m_i = \{M_i, A_k\}$  and  $S_i = \{2M_i\bar{M}_i, A_k\bar{A}_k + \Sigma_k\}$ .

### 3 Grassmannian Taylor Expansion

We demonstrate now how the RG equations for the soft terms appear via Grassmannian Taylor expansion from those for the rigid couplings (11). Indeed, let us substitute eq.(12) into eq.(11) and expand over  $\eta$  and  $\bar{\eta}$ . One has to be careful, however, since as it follows from the soft Lagrangian (4) gauge couplings are involved in chiral Grassmann integrals and expansion over  $\eta$  or  $\bar{\eta}$  makes sense up to F-terms only. On the contrary, the Yukawa couplings  $Y$ , being a product of  $y$  and  $\bar{y}$ , are general superfields, so the expansion is valid for D-terms as well. Having this in mind one gets

$$\dot{\tilde{a}}_i = \tilde{a}_i\gamma_i(\tilde{a}), \quad (13)$$

Consider first the F-terms. Expanding over  $\eta$  one has

$$\dot{a}_i m_i + a_i \dot{m}_i = a_i m_i \gamma_i(a) + a_i \gamma_i(\tilde{a})|_F, \quad (14)$$

or

$$\dot{m}_i = \gamma_i(\tilde{a})|_F = \sum_j a_j \frac{\partial \gamma_i}{\partial a_j} m_j. \quad (15)$$

This is just the RG equation for the soft terms  $M_i$  and  $A_k$  which was written in Ref. [1] in the form

$$\dot{m}_i = D_1 \gamma_i(a). \quad (16)$$

Proceeding the same way for the D-terms one gets after some algebra

$$\dot{S}_i = \gamma_i(\tilde{a})|_D = 2m_i \sum_j a_j \frac{\partial \gamma_i}{\partial a_j} m_j + \sum_j a_j \frac{\partial \gamma_i}{\partial a_j} S_j + \sum_{j,k} a_j a_k \frac{\partial^2 \gamma_i}{\partial a_j \partial a_k} m_j m_k. \quad (17)$$

Substituting  $S_i = m_i \bar{m}_i + \Sigma_i$  one has the RG equation for the mass terms

$$\dot{\Sigma}_i = \sum_j a_j \frac{\partial \gamma_i}{\partial a_j} (m_j m_j + \Sigma_j) + \sum_{j,k} a_j a_k \frac{\partial^2 \gamma_i}{\partial a_j \partial a_k} m_j m_k. \quad (18)$$

One can also obtain the RG equation for the individual soft masses out of field renormalization. Consider the chiral Green function in a rigid theory. It obeys the following RG relation

$$\langle \Phi_i \bar{\Phi}_i \rangle = \langle \Phi_i \bar{\Phi}_i \rangle_0 e^{\int_0^t \gamma_i(a(t')) dt'}. \quad (19)$$

Making the substitution

$$\langle \Phi_i \bar{\Phi}_i \rangle \rightarrow \langle \Phi_i \bar{\Phi}_i \rangle (1 + m_i^2 \eta \bar{\eta}), \quad a \rightarrow \tilde{a},$$

and expanding over  $\eta \bar{\eta}$  (since it stands under the full Grassmann integral only D-term contributes) one has

$$m_i^2 = m_{i0}^2 + \int_0^t dt' \gamma_i(\tilde{a}(t'))|_D. \quad (20)$$

Differentiating this relation with respect to  $t$  leads to

$$\dot{m}_i^2 = D_2 \gamma_i(a), \quad (21)$$

where  $D_2$  stands for a second order differential operator (17) introduced in Ref. [1].

As mentioned above one can make the same expansion not only in equations, but also in solutions. Let us start with the simplest case of pure gauge theory with one gauge coupling. Then one has in a rigid theory

$$\int^\alpha \frac{d\alpha'}{\beta(\alpha')} = \log \left( \frac{Q^2}{\Lambda^2} \right). \quad (22)$$

Making a substitution  $\alpha \rightarrow \tilde{\alpha}$  and  $\tilde{\Lambda} = \Lambda(1 + c\theta^2 + \dots)$  one has

$$\int^{\tilde{\alpha}} \frac{d\alpha'}{\beta(\alpha')} = \log \left( \frac{Q^2}{\tilde{\Lambda}^2} \right). \quad (23)$$

Expansion over  $\eta$  gives

$$M = c\gamma(\alpha), \quad \gamma(\alpha) = \frac{\beta(\alpha)}{\alpha}. \quad (24)$$

One can make the same expansion for any analytic solution in a rigid theory. Below we consider three particular examples, namely the MSSM, the finite SUSY GUT and the Seiberg-Witten N=2 SUSY model.

## 4 Examples

### The MSSM

Consider the MSSM in low  $\tan\beta$  regime. One has three gauge and one Yukawa coupling. The one-loop RG equations are [5]

$$\dot{\alpha}_i = -b_i\alpha_i^2, \quad b_i = (\frac{33}{5}, 1, -3), \quad i = 1, 2, 3, \quad (25)$$

$$\dot{Y}_t = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t), \quad (26)$$

with the initial conditions:  $\alpha_i(0) = \alpha_0$ ,  $Y_t(0) = Y_0$  and  $t = \ln(M_X^2/Q^2)$ . Their solutions are given by [5]

$$\alpha_i(t) = \frac{\alpha_0}{1 + b_i\alpha_0 t}, \quad Y_t(t) = \frac{Y_0 E(t)}{1 + 6Y_0 F(t)}, \quad (27)$$

where

$$E(t) = \prod_i (1 + b_i\alpha_0 t)^{c_i/b_i}, \quad c_i = (\frac{13}{15}, 3, \frac{16}{3}),$$

$$F(t) = \int_0^t E(t') dt'.$$

To get the solutions for the soft terms it is enough to perform substitution  $\alpha \rightarrow \tilde{\alpha}$  and  $Y \rightarrow \tilde{Y}$  and expand over  $\eta$  and  $\bar{\eta}$ . Expanding the gauge coupling in (27) up to  $\eta$  one has (hereafter we assume  $M_{i0} = M_0$ )

$$\alpha_i M_i = \frac{\alpha_0 M_0}{1 + b_i\alpha_0 t} - \frac{\alpha_0 b_i \alpha_0 M_0 t}{(1 + b_i\alpha_0 t)^2} = \frac{\alpha_0}{1 + b_i\alpha_0 t} \frac{M_0}{1 + b_i\alpha_0 t},$$

or

$$M_i(t) = \frac{M_0}{1 + b_i\alpha_0 t}. \quad (28)$$

Performing the same expansion for the Yukawa coupling and using the relations

$$\left. \frac{d\tilde{E}}{d\eta} \right|_{\eta} = M_0 t \frac{dE}{dt}, \quad \left. \frac{d\tilde{F}}{d\eta} \right|_{\eta} = M_0 (tE - F),$$

one finds a well known expression [5]

$$A_t(t) = \frac{A_0}{1 + 6Y_0 F} + M_0 \left( \frac{t}{E} \frac{dE}{dt} - \frac{6Y_0}{1 + 6Y_0 F} (tE - F) \right). \quad (29)$$

To get the solution for the  $\Sigma$  term one has to make expansion over  $\eta$  and  $\bar{\eta}$ . This can be done with the help of the following relations

$$\left. \frac{d^2 \tilde{E}}{d\eta d\bar{\eta}} \right|_{\eta, \bar{\eta}} = M_0^2 \frac{d}{dt} \left( t^2 \frac{dE}{dt} \right), \quad \left. \frac{d^2 \tilde{F}}{d\eta d\bar{\eta}} \right|_{\eta, \bar{\eta}} = M_0^2 t^2 \frac{dE}{dt}.$$

This leads to

$$\Sigma_t(t) = \frac{\Sigma_0 - A_0^2}{1 + 6Y_0 F} + \frac{(A_0 - M_0 6Y_0 (tE - F))^2}{(1 + 6Y_0 F)^2} + M_0^2 \left[ \frac{d}{dt} \left( \frac{t^2}{E} \frac{dE}{dt} \right) - \frac{6Y_0}{1 + 6Y_0 F} t^2 \frac{dE}{dt} \right], \quad (30)$$

which is much simpler than known in the literature [5], though coinciding with it after some cumbersome algebra.

With analytic solutions (29,30) one can analyze asymptotics and, in particular, find the infrared quasi fixed points [6] which correspond to  $Y_0 \rightarrow \infty$

$$Y^{FP} = \frac{E}{6F}, \quad (31)$$

$$A^{FP} = M_0 \left( \frac{t}{E} \frac{dE}{dt} - \frac{tE - F}{F} \right), \quad (32)$$

$$\Sigma^{FP} = M_0^2 \left[ \left( \frac{tE - F}{F} \right)^2 + \frac{d}{dt} \left( \frac{t^2}{E} \frac{dE}{dt} \right) - \frac{t^2}{F} \frac{dE}{dt} \right]. \quad (33)$$

However, the advantage of the Grassmannian expansion procedure is that one can perform it for fixed points as well. Thus the FP solutions (32,33) can be directly obtained from a fixed point for the rigid Yukawa coupling (31) by Grassmannian expansion. This explains, in particular, why fixed point solutions for the soft couplings exist if they exist for the rigid ones and with the same stability properties [7].

## SUSY GUTs

One can consider not only fixed points, but also more complicated configurations like renormalization invariant trajectories which lead to reduction of the couplings [8] or fixed lines or surfaces [9], or finiteness relations [10]. The same procedure is valid here as well.

Let us consider, for example, construction of a finite theory (free from ultraviolet divergences) in the framework of SUSY GUTs. It is achieved in a rigid theory by a proper choice of the field content and of the Yukawa couplings being the functions of the gauge one [10]

$$Y_k = Y_k(\alpha) = c_0^{(k)} \alpha + c_1^{(k)} \alpha^2 + \dots, \quad (34)$$

where coefficients  $c_i^{(k)}$  are calculated within perturbation theory.

To achieve complete finiteness, including the soft terms, one has to choose the latter in a proper way [12]. To find it one just have to modify the finiteness relation for the Yukawa coupling (34) as

$$\tilde{Y}_k = Y_k(\tilde{\alpha}), \quad (35)$$

and expand over  $\eta$  and  $\bar{\eta}$ . This gives:

$$A_k = M \frac{d \ln Y_k}{d \ln \alpha}, \quad (36)$$

and after the rearrangement of terms

$$\Sigma_k = M^2 \frac{d}{d\alpha} \alpha^2 \frac{d \ln Y_k}{d\alpha}, \quad (37)$$

which coincides with the relations found in Ref. [12].

## N=2 SUSY

Consider now the N=2 supersymmetric gauge theory. The Lagrangian written in terms of N=2 superfields is [13]:

$$\mathcal{L} = \frac{1}{4\pi} \text{ImTr} \int d^2\theta d^2\tilde{\theta} \frac{1}{2} \tau \Psi^2, \quad (38)$$

where N=2 chiral superfield  $\Psi(y, \theta, \tilde{\theta})$  is defined by constraints  $\bar{D}_{\dot{\alpha}} \Psi = 0$  and  $\tilde{\bar{D}}_{\dot{\alpha}} \Psi = 0$  and

$$\tau = i \frac{4\pi}{g^2} + \frac{\theta^{\text{topological}}}{2\pi}. \quad (39)$$

The expansion of  $\Psi$  in terms of  $\tilde{\theta}$  can be written as

$$\Psi(y, \theta, \tilde{\theta}) = \Psi^{(1)}(y, \theta) + \sqrt{2} \tilde{\theta}^\alpha \Psi_\alpha^{(2)}(y, \theta) + \tilde{\theta}^\alpha \tilde{\theta}_\alpha \Psi^{(3)}(y, \theta),$$

where  $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} + i\tilde{\theta}\sigma^\mu\bar{\tilde{\theta}}$  and  $\Psi^{(k)}(y, \theta)$  are N=1 chiral superfields.

The soft breaking of N=2 SUSY down to N=1 can be achieved by shifting the imaginary part of  $\tau$ :

$$\tau \rightarrow \tilde{\tau} = \tau + i \frac{4\pi}{g^2} \tilde{\theta}^2 M. \quad (40)$$

This leads to

$$\Delta\mathcal{L} = \frac{1}{g^2} \text{Tr} \int d^2\theta \frac{M}{2} (\Psi^{(1)})^2, \quad (41)$$

which is the usual mass term for N=1 chiral superfield  $\Psi^{(1)}$  normalized to  $1/g^2$ .

Now one can use the power of duality in N=2 SUSY theory and take the Seiberg-Witten solution [14]

$$\tau = \frac{da_D}{du} / \frac{da}{du}, \quad (42)$$



where

$$\begin{aligned} a_D(u) &= \frac{i}{2}(u-1)F(1/2, 1/2, 2; \frac{1-u}{2}), \\ a(u) &= \sqrt{2(1+u)}F(-1/2, 1/2, 1; \frac{2}{1+u}). \end{aligned}$$

Assuming that renormalizations in N=2 SUSY theory follow the properties of those in N=1 one can apply the same expansion procedure. Substituting eq.(40) into (42) with  $u \rightarrow \tilde{u} = u(1 + M_0\tilde{\theta}^2)$  and expanding over  $\tilde{\theta}^2$ , one gets an analog of S-W solution for the mass term:

$$M = M_0 \frac{\mathcal{I}m \left[ u \left( \frac{a_D''}{a_D'} - \frac{a''}{a'} \right) \tau \right]}{\mathcal{I}m \tau}. \quad (43)$$

In perturbative regime ( $u \sim Q^2/\Lambda^2 \rightarrow \infty$ ) one has [13]  $a = \sqrt{2u}$ ,  $a_D = \frac{i}{\pi}a(2 \ln a + 1)$ , which leads to

$$\begin{aligned} \frac{4\pi}{g^2} &= \frac{1}{\pi} [\ln Q^2/\Lambda^2 + 3], \\ M &= M_0 / [\ln Q^2/\Lambda^2 + 3]. \end{aligned}$$

This procedure can be continued introducing soft N=1 SUSY breaking via  $\theta$  dependent  $\tau$  superfield. Thus one can achieve soft SUSY breaking along the chain

$$N = 2 \Rightarrow N = 1 \Rightarrow N = 0$$

preserving the properties of the exact solution. This will lead to a sequence of new solutions for the soft terms like eq.(43).

## 5 Conclusion

We conclude that the Grassmannian expansion in softly broken SUSY theories happens to be a very efficient and powerful method which can be applied in various cases where the renormalization procedure is concerned. It demonstrates once more that softly broken SUSY theories are contained in rigid ones and inherit their renormalization properties.

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